Section 4.2 Applications of Extrema

1) A campground owner has 1000 meters of fencing. He wants to enclose a rectangular field with the fence that he has. Let W represent the width of the field and L represent the length of the field.


1a) Write an equation for the length of the field.

The field is a generic rectangle.

The 1000 meters of fencing represents the perimeter of the field. I need to know the Perimeter of a rectangle formula.

P = 2L + 2W (where L represents the length of the field and W the width)

I can now replace the P with 1000 (I won't put the units in to keep the math easier to follow.)

1000 = 2L + 2W

Part a wants me to solve this equation for L.

I can do this by first dividing everything by 2.

$$\frac{1000}{2} = \frac{2L}{2} + \frac{2W}{2}$$

500 = L + W

Now subtract W from each side to get the answer to part a.

**Answer: L = 500 - W**

1b) Write an equation for the area of the fenced in field.

The area of any rectangle can be found by using the formula:

$A = LW$

I need to replace the L in the formula with 500 – W

$A = (500 - W)W$

$A = W(500 - W)$

**Answer: $A = 500W - W^2$**

1c) Find the domain of the area equation that was created in part b.
(This domain will be of the form: $\# \leq W \leq \#$)

I know the width has to be at least 0 meters as widths can't be negative. I also know that I am building 2 sides that are widths. I have 1000 meters of fencing. If I only built those two sides they could be a maximum of 500 meters each. Then all of my fencing would be used up.

**Answer: Domain $\mathbf{0 \leq W \leq 500}$**

1d) Find the value of w leading to the maximum area.

This is finding the absolute maximum of the function **$A = 500W - W^2$; [0, 500]**

$A' = 500 - 2W$

$500 - 2W = 0$

$500 = 2W$

$250 = W$

| Width | Area | |
|---|---|---|
| 0 | $A = 500(0) - (0)^2 = 0$ | |
| 500 | $A = 500(500) - (500)^2 = 0$ | |
| 250 | $A = 500(250) - (250)^2 = 62500$ | (250, 62500) absolute max |

**Answer: Width = 250 meters**

1e) Find the value of L leading to the maximum area

Use the formula: L = 500 – W

L = 500 – 250

**Answer: L = 250 meters**

1f) Find the maximum area.

Use the formula A = LW with L = 250 and W = 250

A = (250)(250)

**Answer: A = 62,500 square meters**

3) A campground owner has 1000 meters of fencing. He wants to enclose a rectangular field bordering a river, with no fencing needed along the river, and let W represent the width of the field and L represent the length of the field. Make W be the side of the fence that is perpendicular to the river so that two widths and one length will need to be constructed.

See problem 4 in the video for an image.

3a) Write an equation for the length of the field.

We are building 2 Widths and 1 Length.

The perimeter of the sides that we are building are given by the formula:

P = 2W + L

I will replace P with the 1000 meters of fence that is available.

1000 = 2W + L

Now subtract 2W from each side to get the answer.

**Answer: L = 1000 – 2W**

3b) Write an equation for the area of the field.

Even though only three sides are being built. The final region will be a rectangle and the area is still:

A = LW

Now replace the L with 1000 – 2W

A = (1000 – 2W)W

Answer: A = 1000W – $2W^2$

3c) Find the domain of the area equation that was created in part b.
(This domain will be of the form: $\# \leq W \leq \#$)

I know the width has to be at least 0 meters as widths can't be negative. I also know that I am building 2 sides that are widths. I have 1000 meters of fencing. If I only built those two sides they could be a maximum of 500 meters each. Then all of my fencing would be used up.

**Answer: Domain $\mathbf{0 \leq W \leq 500}$**

3d) Find the value of w leading to the maximum area

This is finding the absolute maximum of the function **$A = 1000W - 2W^2$; [0, 500]**

$A' = 1000 - 4W$

$1000 - 4W = 0$

$1000 = 4W$

$250 = W$

| Width | Area | |
|---|---|---|
| 0 | $A = 1000(0) - 2(0)^2 = 0$ | |
| 500 | $A = 1000(500) - 2(1000)^2 = 0$ | |
| 250 | $A = 1000(250) - 2(250)^2 = 125000$ | (250, 125000) absolute max |

**Answer: Width = 250 meters**

3e) Find the value of L leading to the maximum area

Use the formula $L = 1000 - 2W$

$L = 1000 - 2(250)$

$L = 500$

**Answer: Length = 500 meters**

3f) Find the maximum area.

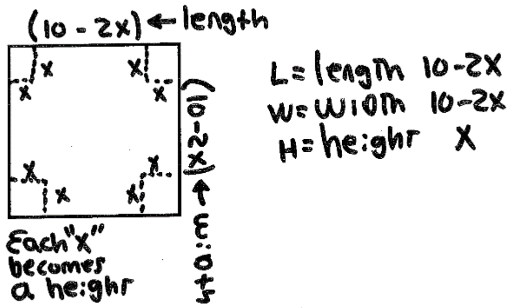
Use the formula $A = LW$ with $L = 500$ meters and $W = 250$ meters

A = (500 meters)(250 meters)

Answer: Area = 125,000 square meters

5) An open box with a square base is to be made from a square piece of cardboard 10 inches on a side by cutting out a square ( x inches by x inches) from each corner and turning up the sides.

5a) Sketch a diagram that models the problem.




5b) Write an equation for the volume of the box.

V = LWH

V = (10 – 2x)(10 – 2x)x

Foil the first two parenthesis to get

V = (100 – 20x – 20x + 4x$^2$)x

V= (100 – 40x + 4x$^2$)x

V = 100x – 40x$^2$ + 4x$^3$

**Answer: V = 4x$^3$ – 40x$^2$ + 100x**

5c) Find the domain of the volume equation created in part b.
(This domain will be of the form: $\# \leq x \leq \#$)

If I don't make a cut x will be equal to 0. I need to make two equal cuts on each side. Each side is only 10 inches, so the cuts can't be more than half of 10 inches. 5 inches is the maximum cut.

**Answer:** $\mathbf{0 \leq x \leq 5}$

5d) Find the value of x that makes the volume the largest.

$V' = 12x^2 - 80x + 100$

$12x^2 - 80x + 100 = 0$

I will use the quadratic formula to solve for this.

$x = \frac{-(-80) \pm \sqrt{(-80)^2 - 4(12)(100)}}{2(12)}$ I used my calculator to get the values of x.

x = 5 and x = 5/3

Now I will make my table with these and the domain values.

| x | Volume | point |
|---|---|---|
| 0 | $V = 4(0)^3 - 40(0)^2 + 100(0) = 0$ | (0, 0) |
| 5 | $V = 4(5)^3 - 40(5)^2 + 100(5) = 0$ | (5,0) |
| 5/3 | $V = 4(5/3)^3 - 40(5/3)^2 + 100(5/3) = 74.07 = 2000/27$ | (5/3, 74.07) Abs Max |

**Answer: x = 5/3 inches**

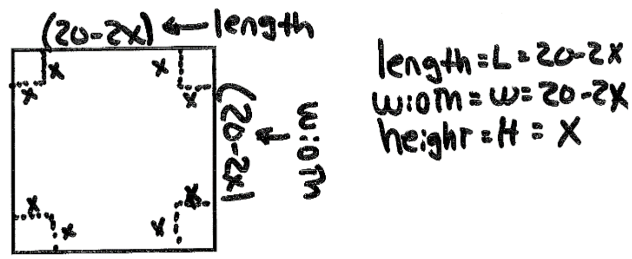
5e) Find the maximum volume.

The y-coordinate of the absolute maximum is what I need.

**Answer: Volume = 74.07 cubic inches or 74.07 cubic inches**

7) An open box is to be made by cutting a square corner of a 20 inch by 20 inch piece of metal then folding up the sides. What size square should be cut from each corner to maximize volume?

7a) Sketch a diagram that models the problem.




7b) Write an equation for the volume of the box.

V = LWH

V = (20 – 2x)(20 – 2x)x

Foil the first two parenthesis to get

V = (400 – 40x – 40x + 4x$^2$)x

V= (400 – 80x + 4x$^2$)x

V = 400x – 80x$^2$ + 4x$^3$

**Answer: V = 4x$^3$ – 80x$^2$ + 400x**

7c) Find the domain of the volume equation created in part b.
(This domain will be of the form: $\# \leq x \leq \#$)

If I don't make a cut x will be equal to 0. I need to make two equal cuts on each side. Each side is only 10 inches, so the cuts can't be more than half of 20 inches. 10 inches is the maximum cut.

**Answer: $\mathbf{0 \leq x \leq 10}$**

7d) Find the value of x that makes the volume the largest.

$V' = 12x^2 - 160x + 400$

$12x^2 - 160x + 400 = 0$

I will use the quadratic formula, and press buttons on my calculator to solve for x.

$$x = \frac{-(-160)\pm\sqrt{(-160)^2-4(12)(400)}}{2(12)}$$

This gives x = 10, 10/3

Now I will make my table with these and the domain values.

| x | Volume | point |
|---|---|---|
| 0 | $V = 4(0)^3 - 80(0)^2 + 400(0) = 0$ | (0,0) |
| 10 | $V = 4(10)^3 - 80(10)^2 + 400(10) = 0$ | (10,0) |
| 10/3 | $V = 4(10/3)^3 - 80(10/3)^2 + 400(10/3) = 592.59 = 16000/27$ | (10/3, 16000/27) Abs max |

Answer: x = 10/3 inches

7e) Find the maximum volume.

The y-coordinate of the absolute maximum is what I need.

Volume = 592.59 cubic inches or 16000/27 cubic inches